Prediction of Long Term Stability by Extrapolation

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Contents

Abstract

This paper studies the possibility of using the survival function, to predict long term stability by extrapolation. The survival function is a function of the initial coordinates and is the number of turns a particle will survive for a given set of initial coordinates. To determine the difficulties in extrapolating the survival function, tracking studies were done to compute the survival function. The survival function was found to have two properties that may cause difficulties in extrapolating the survival function. One is the existence of rapid oscillations, and the second is the existence of plateaus. It was found that it appears possible to extrapolate the survival function to estimate long term stability by taking the two difficulties into account. A model is proposed which pictures the survival function to be a series of plateaus with rapid oscillations superimposed on the plateaus. The tracking studies give results for the widths of these plateaus and for the seperation between adjacent plateaus which can be used to extrapolate and estimate the location of plateaus that indicate survival for longer times than can be found by tracking.

Chapter 1

Introduction

This paper studies the possibility of using the survival function, to predict long term stability by extrapolation [1, 2, 3, 4, 5, 6, 7]. The survival function is a function of the initial coordinates and is the number of turns a particle will survive for a given set of initial coordinates. To determine the difficulties in extrapolating the survival function, tracking studies were done to compute the survival function. The survival function was found to have two properties that may cause difficulties in extrapolating the survival turns function. One is the existence of rapid oscillations, and the second is the existence of plateaus.It was found that it appears possible to extrapolate the survival function to estimate long term stability by taking the two difficulties into account.

A model is proposed which pictures the survival function to be a series of plateaus with rapid oscillations superimposed on the plateaus. The tracking studies give results for the widths of these plateaus and for the seperation between adjacent plateaus which can be used to extrapolate and estimate the location of plateaus that indicate survival for longer times than can be found by tracking.

Chapter 2

The survival function, j_{trns}.

For a given set of initial coordinate, x_0, p_{x0}, y_0, p_{y0}, one can find by tracking the survival time in turns, which is the number of turns the particle will survive before becoming unstable and which will be denoted by j_{trns}. This determines the function $j_{trns}(x_0, p_{x0}, y_0, p_{y0})$ which will be called the survival function [8, 9, 10, 11]. If one limits the tracking to 1.0 10^6 turns or less, one can find j_{trns} for those x_0, p_{x0}, y_0, p_{y0} for which j_{trns} is less than or equal to 1.0 10^6.

A tracking study was done of particle motion with no rf present, using an older version of the RHIC lattice. Random and systematic field error multipoles are present up to order 10. The particle momemtum is $dp/p = 0$. As the first case studied, the initial coordinates x_0, p_{x0}, y_0, p_{y0} are chosen along the direction in phase space given by $p_{x0} = 0, p_{y0} = 0$ and $\epsilon_{x0} = \epsilon_{y0}$, where $\epsilon_{x0}, \epsilon_{y0}$ are the linear emittances in the absence of the error multipoles. Along this direction in phase space , j_{trns} may be considered to be a function of x_0. For a given initial coordinate, x_0, one can find by tracking the survival time in turns, which is the number of turns the particle will survive before becoming unstable and will be denoted by j_{trns}. This determines the survival function [8, 9, 10, 11] $j_{trns}(x_0)$. If one limits the tracking to 1 10^6 turns or less, one can find j_{trns} for those x_0 for which j_{trns} is less than or equal to 1 10^6. For $dp/p = 0$, it is assumed that on the closed orbit $x = 0$ and x_0 is also the initial betatron oscillation amplitude.

The tracking was first done using x_0 which are seperated by $dx_0 = .1$ mm. The results for j_{trns} versus x_0 are shown in Fig. 2.1. The results in this figure may be looked at as the results of a search in x_0 starting at large x_0 and decreasing x_0 in steps of $dx_0 = .1mm$. The figure shows an apparent

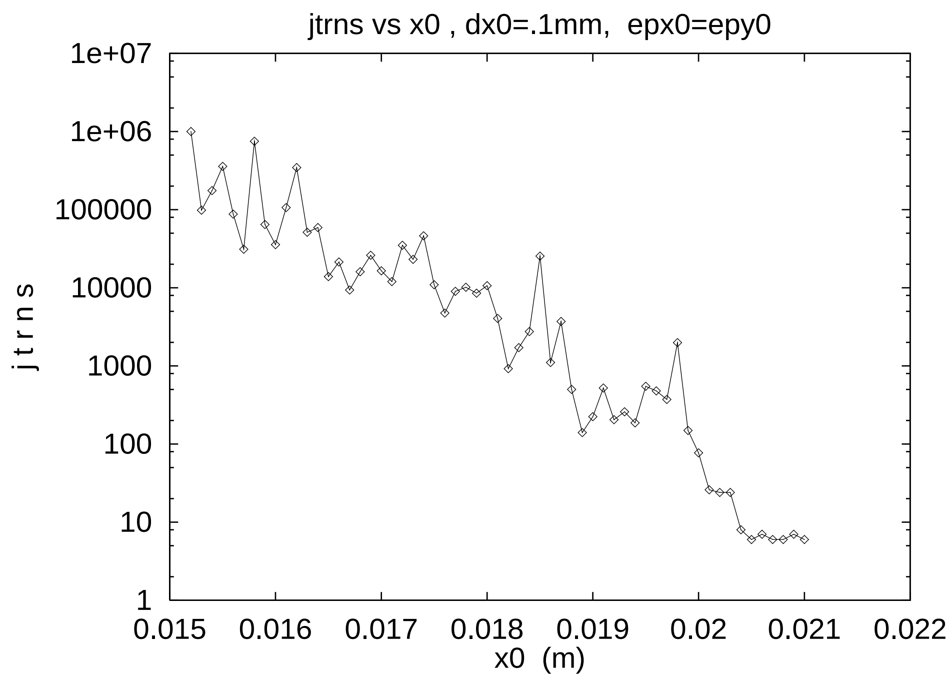

Figure 2.1: j_{trns} versus x_0. $dp/p = 0$, $p_{x0} = 0, p_{y0} = 0, \epsilon_{x0} = \epsilon_{y0}$ direction, dx_0=.1mm. In the figure, jtrns, x0 , dx0, epx0, epy0 represent j_{trns}, x_0, dx_0 ,ϵ_{x0}, ϵ_{y0}.

stability limit for 1e6 turns of u_{sl} =15.2 mm.

2.1 Rapid oscillations in the survival function

Fig. 2.1 shows rapid oscillations in j_{trns} with x_0. Large changes in j_{trns} occur when x_0 changes by .1 mm. The oscillations extend over about .3 mm. Tracking studies show that the oscillations become more rapid when the search interval, dx_0, is decreased. This is indicated in Fig. 2.2 where dx_0 is decreased to .05 mm and in Fig. 2.3 where dx_0 is decreased to .025 mm. Results for different search intervals, dx_0, are shown in Table 2.1. dx_0 is decreased from .1mm to .0001 mm . The wavelength of the oscilations , Δx_0, as measured from the stability limit for 1e6 turns, u_{sl}, to the location of the first peak in j_{trns} decreases from about .3 mm to .0002 mm. Also listed for each value of dx_0 are the apparent stability limit for 1e6 turns, u_{sl}, and j_{trns}

dx_0 (mm)	Δx_0 (mm)	u_{sl} (mm)	j_{trns} at $u_{sl} + dx_0$
.1	.3	15.2	100000
.01	.06	15.27	150000
.001	.003	15,270	82605
.0001	.0002	15.2793	60693

Table 2.1: Results for different search intervals, dx_0. dx_0 is decreased from .1mm to .0001 mm . Δx_0 is the wavelength of the oscilations , as measured from the stability limit for 1e6 turns, u_{sl}, to the location of the first peak in j_{trns} . Also listed for each value of dx_0 are the apparent stability limit for 1e6 turns, u_{sl}, and j_{trns} at $u_{sl} + dx_0$.

at $u_{sl} + dx_0$.

Table 2.1 shows that the wavelength of the oscilations , Δx_0 is roughly proportional to the size of the search interval, dx_0. The value of j_{trns} at $u_{sl} + dx_0$ shows that near u_{sl}, j_{trns} changes appreciably in the small change in x_0 given by dx_0. Tracking results show that this seems to hold even at extremely small dx_0.The computed results appear to indicate that $j_{trns}(x_0)$ is not a continuous function of x_0. For a continuous function of x_0, one can find a small enough interval in x_0 such that the differnce between the values of the function for any two x_0 in that interval is very small. This does not appear to be true for $j_{trns}(x_0)$.

The existence of the rapid oscillations in the survival function, $j_{trns}(x_0)$, would seem to make it difficult to extrapolate to find those x_0 that survive for more than 1e6 turns. However one could view the survival function shown in Fig. 2.1 as being made up of the rapid oscillations superimposed on a smoother, more slowly varying function which could be used for the extrapolation. This is discussed further in section 2.3.

2.2 Plateaus in the survival function

Looking at Fig. 2.1, one can make out plateaus in the survival function, $j_{trns}(x_0)$. The plateaus are regions where j_{trns} oscillates rapidly around an almost constant value of j_{trns}. The plateaus can be seen somewhat more clearly if one reduces the search interval dx_0, as shown in Fig. 2.2 where dx_0 is decreased to $dx_0 = .05mm$. One can make out 4 plateaus located at about j_{trns}=1.5e5, 2e4, 3500, 400 turns. Possible plateaus with j_{trns} less than 100

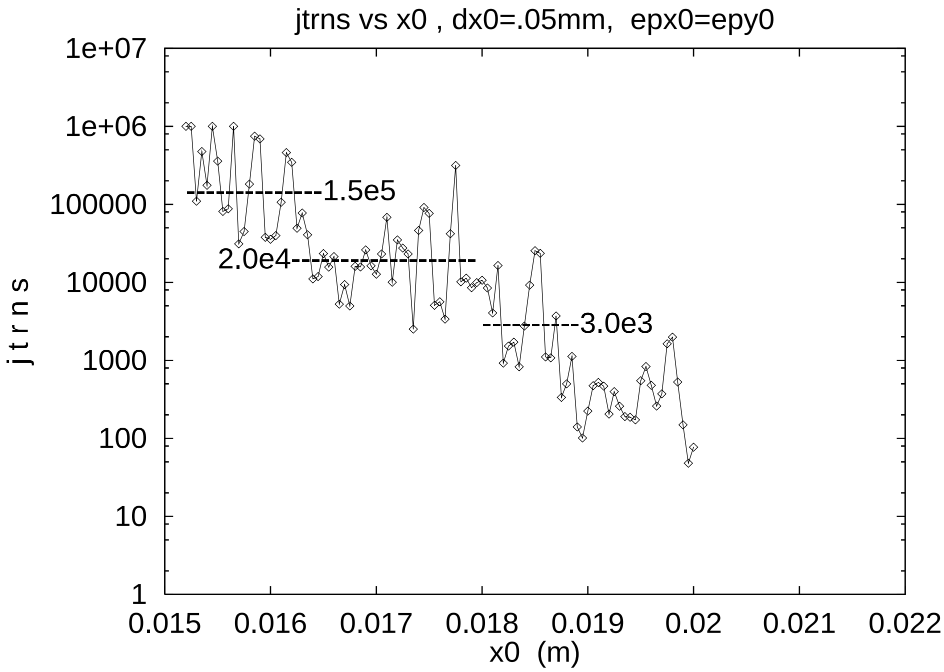

Figure 2.2: j_{trns} versus x_0. $dp/p = 0$, $p_{x0} = 0, p_{y0} = 0, \epsilon_{x0} = \epsilon_{y0}$ direction, dx_0=.05mm. In the figure, jtrns, x0 , dx0, epx0, epy0 represent j_{trns}, x_0, dx_0 ,ϵ_{x0}, ϵ_{y0}.

turns are being ignored. It will be seen that the width of the plateaus do not depend on the search interval, dx_0. This is also true of the location of the plateaus in j_{trns}. This is shown in Fig. 2.3 where dx_0=.025 mm.

The existence of the plateaus in the survival function, $j_{trns}(x_0)$, would seem to make it difficult to extrapolate to find those x_0 that survive for more than 1e6 turns. If one does the extrapolation using points which are close to the stability limit for 1e6 turns, u_{sl}, one may get incorrect results as these points may lie on one of the plateaus.

2.3 Extrapolation of the survival function

The data given above leads to a model of the survival functioon, which pictures it as sequence of plateaus. Within the plateaus, j_{trns} oscillates about some constant vaue of j_{trns} which will be called the level of the plateau. The

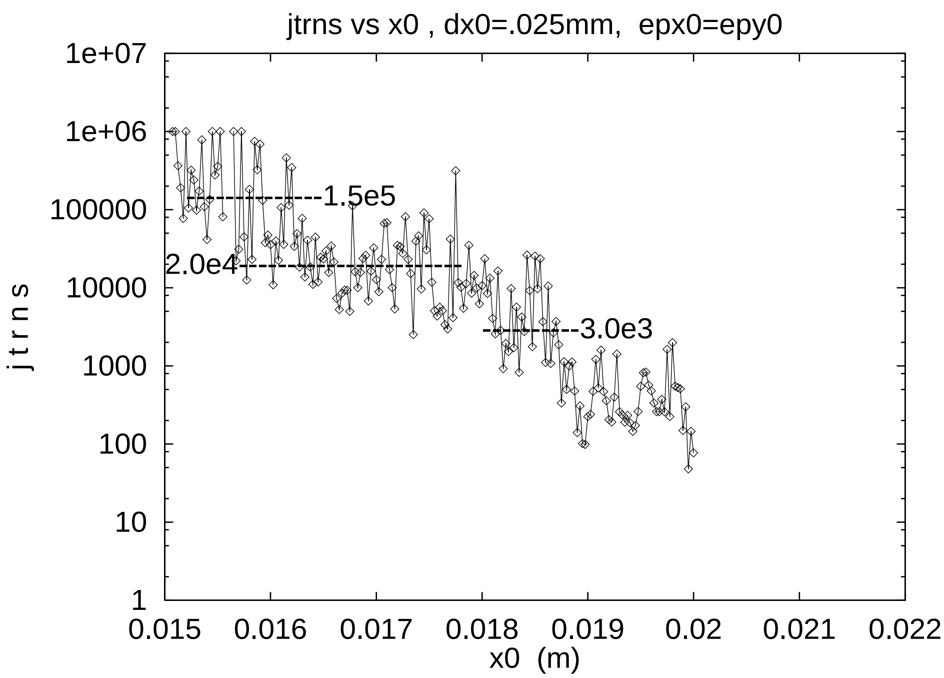

Figure 2.3: j_{trns} versus x_0. $dp/p = 0$, $p_{x0} = 0, p_{y0} = 0, \epsilon_{x0} = \epsilon_{y0}$ direction, dx_0=.025mm. In the figure, jtrns, x0 , dx0, epx0, epy0 represent j_{trns}, x_0, dx_0 ,ϵ_{x0}, ϵ_{y0}.

existence of the plateaus makes extrapolation of the survival function appear difficult. The last plateau that was measured has a level of about 1.5e5 turns and a width of about 1.2 mm. An interesting question is what is the level of the next plateau, at lower x_0, after the last plateau that was measured. In the following this plateau will be referred to as the 'next plateau'.

Some help with the extrapolation is also provided by plotting $1/Log(j_{trns})$ against x_0 as shown in Fig. 2.4, where the search interval is dx_0=.1mm and points with j_{trns} less than 400 turns are omitted. It will be seen below that the seperation between adjacent plateaus does not vary greatly when measured as the change in $1/Log(j_{trns})$.

To help locate the 'next plateau', long runs were done starting with x_0=15.2mm, and decreasing x_0 1n steps of .1mm. In order to detect the beginning of the 'next plateau', the runs have to be long enough not to be confused by the rapid oscillations in j_{trns} that occur within each plateau. Runs of length 2e7 turns were chosen, and these runs take about 10 days for

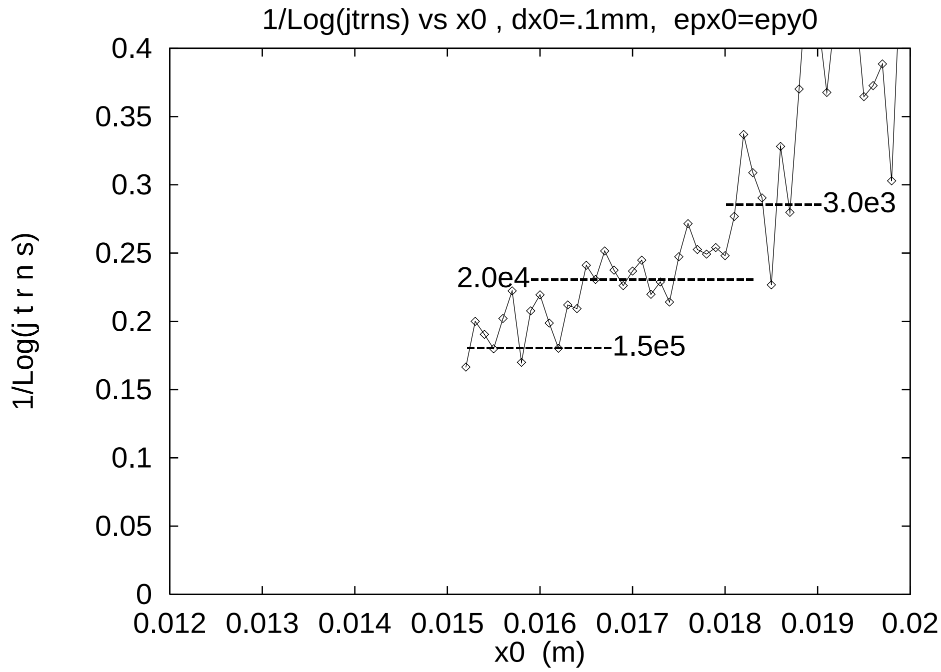

Figure 2.4: $1/Log(j_{trns})$ versus x_0. $dp/p = 0$, $p_{x0} = 0, p_{y0} = 0, \epsilon_{x0} = \epsilon_{y0}$ direction, dx_0=.1mm. In the figure, Log(jtrns), x0 , dx0, epx0, epy0 represent $1/Log(j_{trns})$, x_0, dx_0 ,ϵ_{x0}, ϵ_{y0}.

the RHIC lattice used.

The results are shown in Fig. 2.5 . One sees from this figure that the 'next plateau' appears to start at x_0=15.3 mm and ends at about 14.0 mm, and has a level of about j_{trns}=15e6 turns and a width of 1.2mm . The previous plateau goes from 16.5mm to 15.2 mm and has a width of 1.3mm and a level of jtrns=1.5e5 turns. The width of the plateau is measured here from the end of one plateau in x0 to the end of the adjacent plateau and includes the transition region where the points move from one plateau to the next. The width of the 'next plateau' is difficult to measure, as the adjacent plateau at still lower x_0 is estimated to have a level of 3e9 turns, and cannot be found by tracking. One can see that the width is larger than 1.2mm. The x_0 at 14.0 mm survived more than 89.9e6 turns. The data given above for the 'next plateau' is somewhat in error but it seems better to use it than to throw away this information.

The results are also shown as a $1/Log(j_{trns})$ versus x_0 plot in Fig. 2.6.

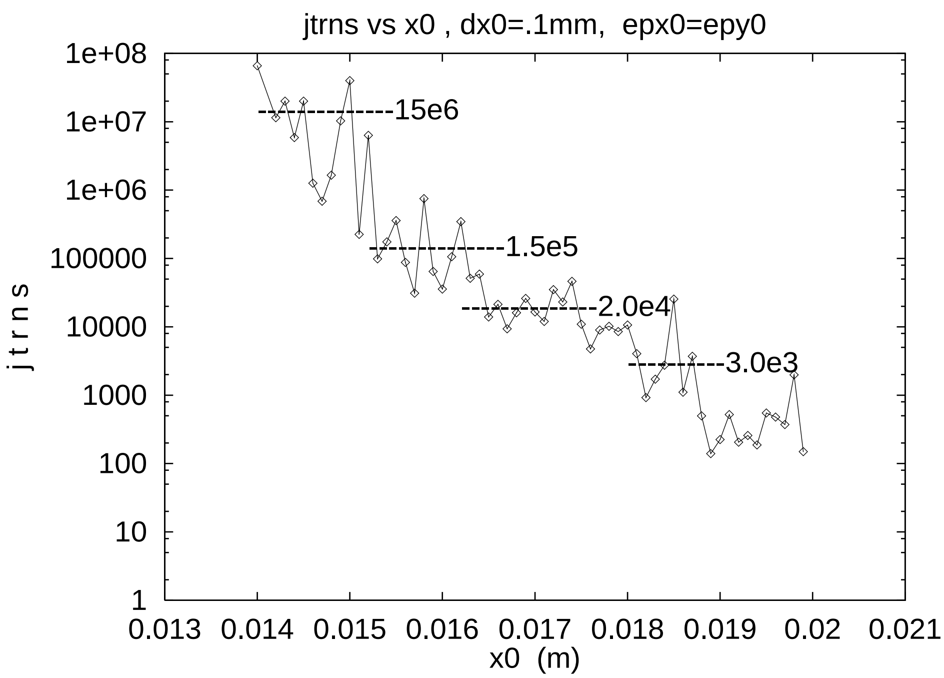

Figure 2.5: j_{trns} versus x_0 including points with j_{trns} up to 4e7. $dp/p = 0$, $p_{x0} = 0, p_{y0} = 0, \epsilon_{x0} = \epsilon_{y0}$ direction, dx_0=.1mm. In the figure, jtrns, x0 , dx0, epx0, epy0 represent j_{trns}, x_0, dx_0 ,ϵ_{x0}, ϵ_{y0}.

One can see three plateaus near the stability boundary at the levels of about jtrns= 15e6, 1.5e5,and 2e4 turns. The properties of the plateaus are summarized in Table 2.2. The seperation between the plateaus in the $1/Log(j_{trns})$ plot are given by .054 and .039, so that the seperation between the plateaus in the $1/Log(j_{trns})$ are not too different. The widths of the plateaus that were measured were 1.2+, 1.3 and 1.7 mm.

The plateau model will now be used to extrapolate and investigate long term stability in RHIC. In RHIC, j_{trns}=1e9 turns coresponds to a survival

plateau level, j_{trns}	15e6	1.5e5	2e4
plateau level,$1/Log(j_{trns})$	.139	.193	.232
plateal seperation in $1/Log(j_{trns})$	.054	.039	—
plateau width, Δx_0 (mm)	1.2+	1.3	1.7

Table 2.2: Plateau parameters for the epx0=epy0 direction.

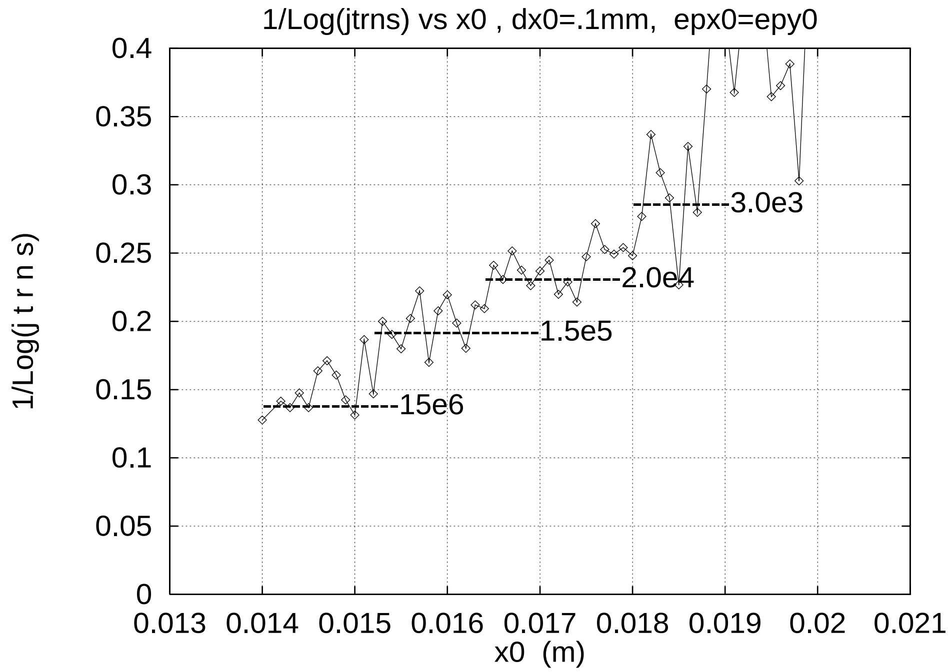

Figure 2.6: $1/Log(j_{trns})$ versus x_0 showing the next plateau and the last measureed plateau. $dp/p = 0$, $p_{x0} = 0, p_{y0} = 0, \epsilon_{x0} = \epsilon_{y0}$ direction, dx_0=.1mm. In the figure, 1/Log(jtrns), x0 , dx0, epx0, epy0 represent $1/Log(j_{trns})$, x_0, dx_0 ,ϵ_{x0}, ϵ_{y0}.

time of about 3.5 hours. The data given above will be used to extrapolate to find the plateau whose level is greater than or equal to j_{trns}=1e9 turns. This plateau will be called the 1e9 plateau. The existence of the plateaus indicates that there are limits on the accuracy one can hope to achieve by extrapolation. The two parameters one needs to extrapolate the survival function are the plateau width and the plateau level seperation. One cannot be certain what these parameters will be at j_{trns} of the order of 1e9 turns. However, one can use the data found for these two parameters at j_{trns} of the order of 1e6 turns , to make the best estimate for these parameters at j_{trns} of the order of 1e9 turns.

The plateau width and the plateau level seperation were studied for three different cases coresponding to three different directions in phase space. The results are summarized in chapter 5. The widths of the plateaus were found to be roughly constant when measured in terms of X_0=$[\beta_{x0}(\epsilon_{x0} + \epsilon_{y0})]^{.5}$ with

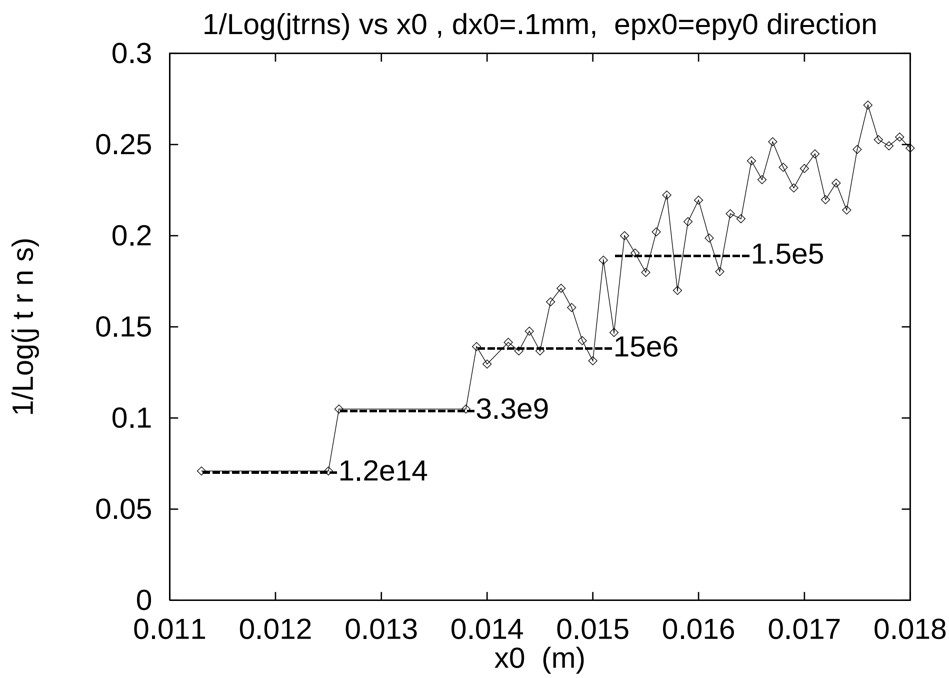

Figure 2.7: $1/Log(j_{trns})$ versus x_0 showing the plateaus found by extrapolation, including the 1e9 plateau. $dp/p = 0$, $p_{x0} = 0, p_{y0} = 0, \epsilon_{x0} = \epsilon_{y0}$ direction, dx_0=.1mm. In the figure, 1/Log(jtrns), x0 , dx0, epx0, epy0 represent $1/Log(j_{trns})$, x_0, dx_0 ,ϵ_{x0}, ϵ_{y0}.

an average value ΔX_0=2.00 mm. The plateau level seperation measured in $1/Log(j_{trns})$ varied from .054 to .019 with an average value of .033. It is suggested that in the extrapolation, the average value of these two parameters be used. For the case being considered here, it is assumed in the extrapolation that the plateau widths in x_0 will be 1.4 mm corresponding to the average value of ΔX_0=2.00 mm, and the plateau level seperation is .033 in $1/Log(j_{trns})$. Note that in this case X_0=1.414 x_0

The results found using these assumptions are shown in Fig. 2.7, where two new plateaus are shown that were found by extrapolation. The plateau just below the next plateau in x0 has the level of j_{trns}=3.3e9 turns and is the 1e9 plateau in this example. The 1e9 plateau goes from x_0=13.9mm to x_0=12.6mm. The aperture for 1e9 turns may be taken as about 13.9mm, which may be compared to the 15.2 mm found using runs of 1e6 turns. This indicates a loss of about 1.3mm or about 9% due to the required survival

time of 1e9 turns. One may note that the level of the 1e9 plateau being 3.3e9 turns, one may expect that some of the x_0 on this plateau will not survive 1e9 turns due to the oscillations in jtrns that will occur on this plateau.To be safer one could assume the aperture for 1e9 turns to be 12.6 mm, which is the beginning of the adjacent plateau with a level of 1.2e14 turns, giving a loss of 17%.

The procedure used indicates that in this case the result is not sensitive to changes in the two assumptions made regarding the width and level seperation of the plateaus. If the plateau model continues to hold for the extrapolated plateaus, and the width and level seperation remain roughly 1.3 mm and .034 respectively, then the result for the aperture for 1e9 turns will be about the same.

The plateau model developed above avoids certain problems that arise in tracking studies. In trying to find the aperture for the survival time of 1e9 turns, the more usual approach is to try find the x_0 such that all smaller x_0 will survive for 1e9 turns. If while doing a tracking search starting from large x_0, one finds a x_0 that survives for 1e9 turn, then one has to ask whether all smaller x_0 will survive for 1e9 turns. This is a difficult question to answer. It is even possible, that there are are always smaller x_0 that will not survive 1e9 turns, although these x_0 may become very scarce at smaller x_0. In the plateau model, in trying to find the aperture for the survival time of 1e9 turns, the approach is to try to find the plateau whose level is greater than 1e9 turns. This is a better defined target, and the expectation is that this plateau will indicate a region of x_0 where most x_0 will survive 1e9 turns.

Chapter 3

Survival function along the $\epsilon_{y0} = 0$ direction

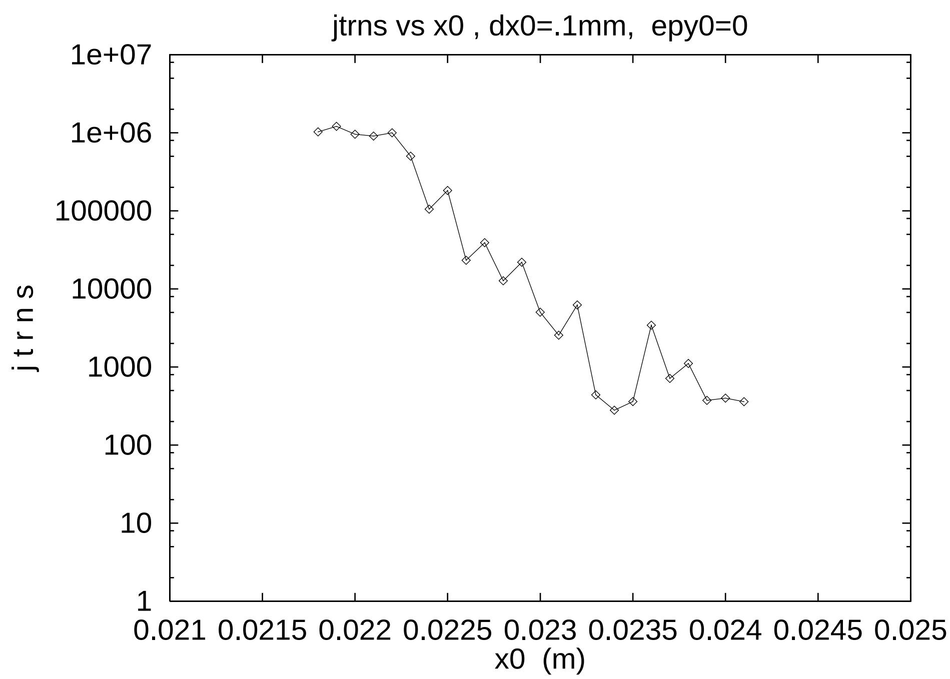

Figure 3.1: j_{trns} versus x_0 . $dp/p = 0$, $p_{x0} = 0, p_{y0} = 0, \epsilon_{y0} = 0$ direction, dx_0=.1mm. In the figure, jtrns, x0 , dx0, epx0, epy0 represent j_{trns}, x_0, dx_0 ,ϵ_{x0}, ϵ_{y0}.

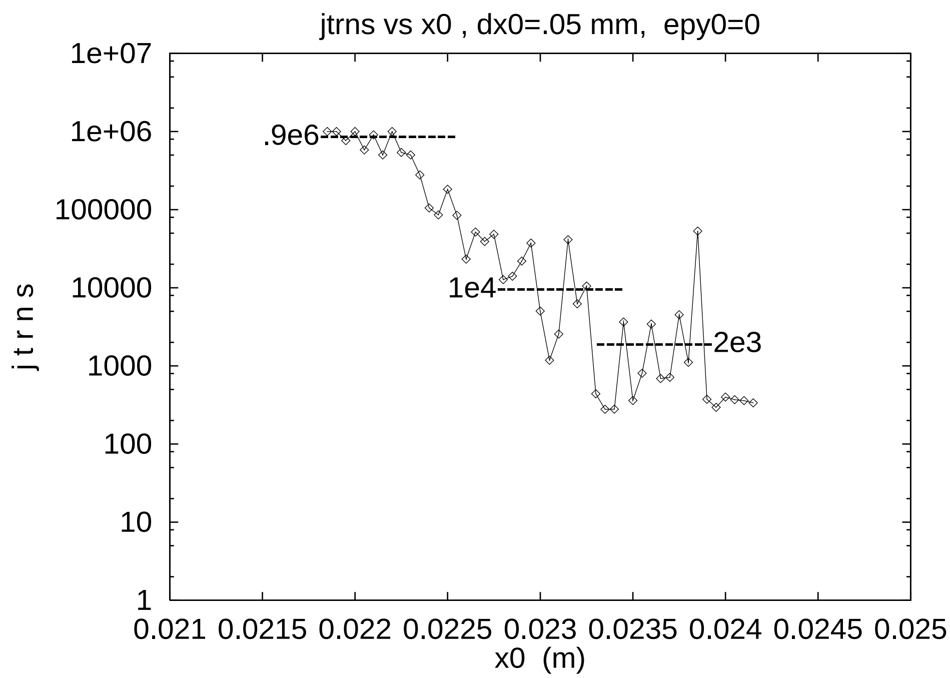

Figure 3.2: j_{trns} versus x_0 . $dp/p = 0$, $p_{x0} = 0, p_{y0} = 0, \epsilon_{y0} = 0$ direction, dx_0=.05mm. In the figure, jtrns, x0 , dx0, epx0, epy0 represent j_{trns}, x_0, dx_0 ,ϵ_{x0}, ϵ_{y0}.

The tracking results presented above were all done along the $\epsilon_{x0} = \epsilon_{y0}$ direction. Results will now be given for the $y_0 = 0, p_{y0} = 0$ or $\epsilon_{y0} = 0$ direction. Tracking studies of other cases will test the consistancy of the plateau model. The direction in the space of x_0, p_{x0}, y_0, p_{y0} is further defined by $p_{x0} = 0$. The particle motion is 4 dimensional because of the presence of skew multipoles. j_{trns} may then be considered to be a function of x_0. Fig. 3.1 shows j_{trns} as a function of x_0 as found with tracking runs of 1e6 turns. The apparent stability limit using 1e6 turns and dx_0=.1mm is 22.0 mm. To make the plateaus more visible , one can reduce the search interval dx_0 to dx_0=.05 mm. These results are shown in Fig. 3.2. One sees that there are two plateaus in the region shown with j_{trns} greater than or equal to 1e4, whose levels are located at j_{trns}= .9e6 , and 1e4 turns .The oscillations on the plateau near the stability boundary, appear to be smaller than those seen in the $\epsilon_{x0} = \epsilon_{y0}$ case.

Runs of about 2e7 turns were done to find the shape of the 'next plateau'

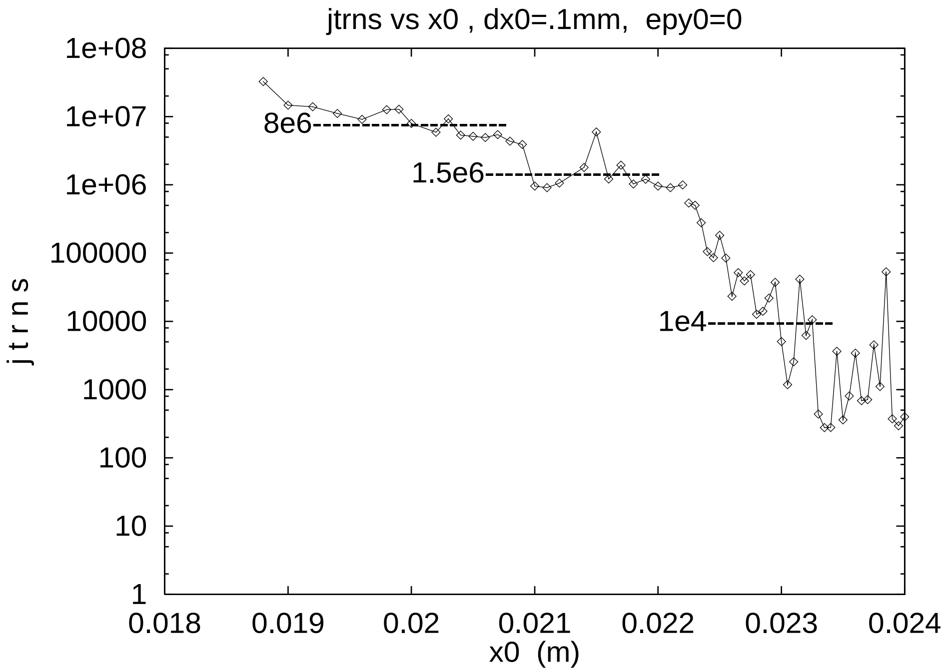

Figure 3.3: j_{trns} versus x_0 including points with j_{trns} up to 4e7. $dp/p = 0$, $p_{x0} = 0, p_{y0} = 0, \epsilon_{y0} = 0$ direction, dx_0=.1 mm with dx_0=.05 mm at larger x_0. In the figure, jtrns, x0 , dx0, epx0, epy0 represent j_{trns}, x_0, dx_0 ,ϵ_{x0}, ϵ_{y0}.

in the region where x_0 is less than or equal to 21.0 mm. Enough tracking runs were done to determine the beginning , and the level of the 'next plateau'. The results are shown in Fig. 3.3. Including these longer runs , one sees that the 'next plateau' begins at x0=21.0mm and the level of the 'next plateau' is about j_{trns}=8e6 turns. Here, the begining of the plateau is the edge at larger x_0 and the end is the edge at lower x_0. Th end of the 'next plateau' is some what difficult to determine, as the adjacent plateau at lower x_0 has a high level of about 1e9 turns. The end of the ' next plateau' was taken to be at x_0=19.0 mm. The adjacent plateau at higher x0 is at the level of 1.5e6 turns and with the width of 1.8 mm. These two plateaus are seperated by .017 in $1/Log(j_{trns})$ which is smaller than the .054 found in the ϵ_{x0}=ϵ_{y0} case. The results are also shown as $1/Log(j_{trns})$ versus x_0 plot in Fig. 3.4.

The data found for these two plateaus will be used to extrapolate and find the 1e9 plateau in this case. In the extrapolation, the results found in chapter 5 for the average plateau width and level seperation will be used. In

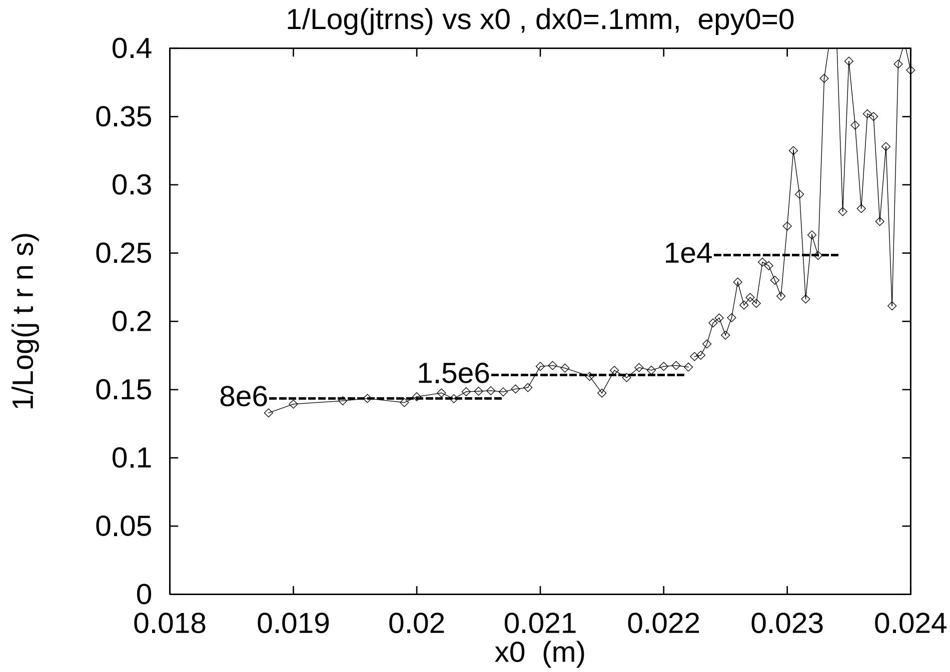

Figure 3.4: $1/Log(j_{trns})$ versus x_0 . $dp/p = 0$, $p_{x0} = 0, p_{y0} = 0, \epsilon_{y0} = 0$ direction, dx_0=.1 mm. In the figure, Log(jtrns), x0 , dx0, epx0, epy0 represent $1/Log(j_{trns})$, x_0, dx_0 ,ϵ_{x0}, ϵ_{y0}.

this case, this leads to the assumptions that each plateau is 2.0mm wide and a plateau level seperation of .033 in $1/Log(j_{trns})$ will be used here.

The results are shown in Fig. 3.5, where two extrapolated plateaus are shown with the plateau levels 8.5e8, and 4.5e12 . The aperture for 1e9 turns was taken to be 17.1mm, the beginning of the plateau with a level of 4.5e12 turns. 17.1 mm is to be compared with the aperture of 21.9 mm found with runs of 1e6 turns. A loss of 4.8 mm or 22%.

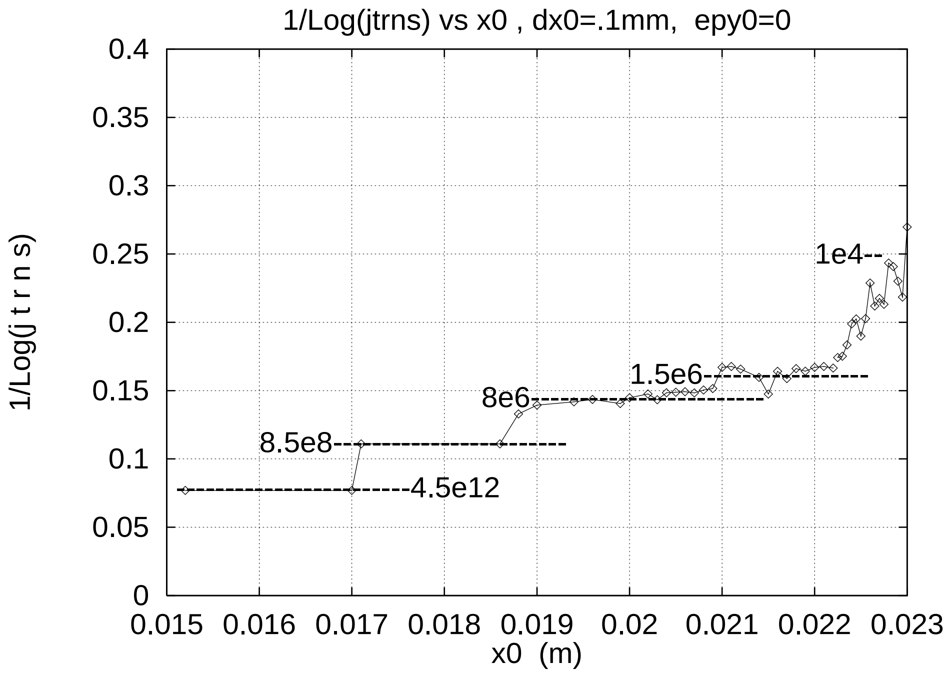

Figure 3.5: $1/Log(j_{trns})$ versus x_0 showing the plateaus found by extrapolation, including the 1e9 plateau. $dp/p = 0$, $p_{x0} = 0, p_{y0} = 0, \epsilon_{x0} = \epsilon_{y0}$ direction, dx_0=.1mm. In the figure, 1/Log(jtrns), x0 , dx0, epx0, epy0 represent $1/Log(j_{trns})$, x_0, dx_0 ,ϵ_{x0}, ϵ_{y0}.

Chapter 4

Survival function along $\epsilon_{x0} = 0$ direction

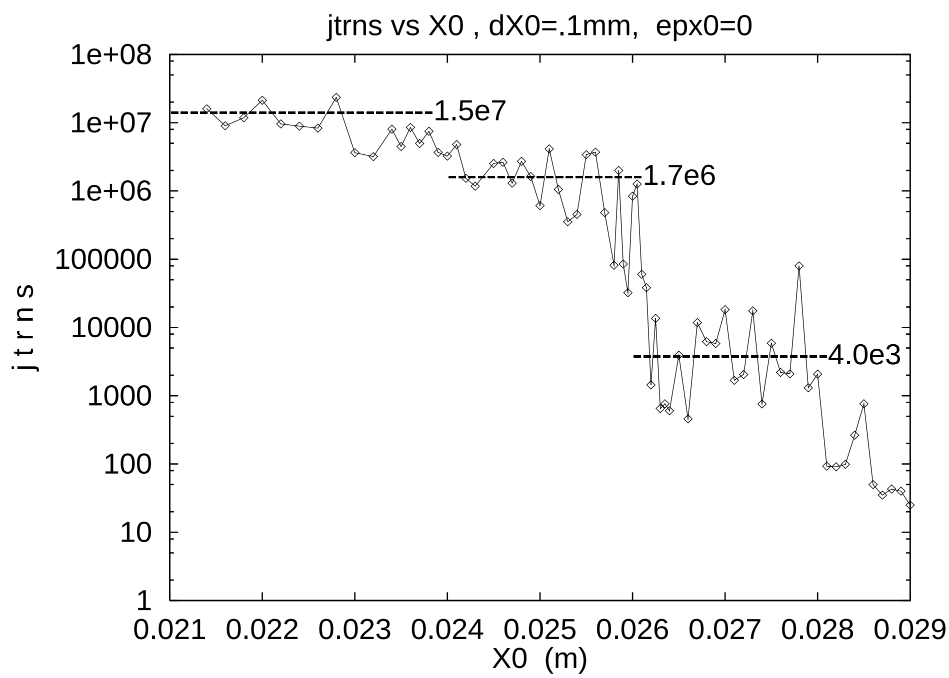

Figure 4.1: j_{trns} versus X_0 .$X_0 = (\beta_{x0}/\beta_{y0})^{.5} y_0$. $dp/p = 0$, x_0=0, $p_{x0} = 0, p_{y0} = 0, \epsilon_{x0} = 0$ direction, dx_0=.1 mm and runs go up to 2e7 turns. In the figure, jtrns, X0 , dX0, epx0, epy0 represent j_{trns}, X_0, dX_0 ,ϵ_{x0}, ϵ_{y0}.

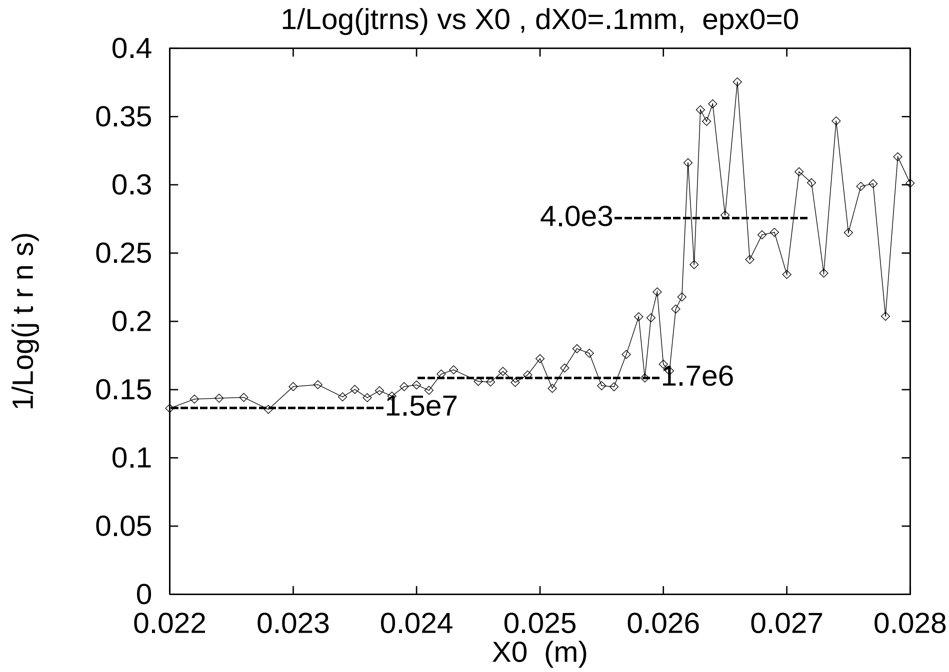

Figure 4.2: $1/Log(j_{trns})$ versus X_0 . $dp/p = 0$, $p_{x0} = 0, p_{y0} = 0, \epsilon_{x0} = 0$ direction, dX_0=.1 mm. $X_0 = (bex0/bey0)^{.5} y_0$. In the figure, 1/ Log(jtrns), X0 , dX0, epx0, epy0 represent $1/Log(j_{trns})$, X_0, dX_0 ,ϵ_{x0}, ϵ_{y0}.

Results will now be given for the $x_0 = 0, p_{x0} = 0$ or $\epsilon_{x0} = 0$ direction.In order to be able to compare results with those of the above two cases, j_{trns} will plotted against $X_0 = (\beta_{x0}/\beta_{y0})^{.5} y_0$. Fig. 4.1 shows j_{trns} as a function of X_0 as found with tracking runs of about 2e7 turns.The apparent stability limit using 1e6 turns and dx_0=.1mm is u_{sl}=24.9 mm. In Fig. 4.1 one can make out two plateaus with levels larger than 1e4 turns. The levels of these two plateaus are at 1.7e6 turns and 1.5e7 turns. The end of the 1.5e7 plateau was taken as X_0=21.4 mm. The results are also shown as a $1/Log(j_{trns})$ plot in Fig. 4.2.

The data found for these two plateaus will be used to extrapolate and find the 1e9 plateau in this case. In the extrapolation, the results found in chapter 5 for the average plateau width and level seperation will be used. In this case, this leads to the assumptions that each plateau is 2.0 mm wide and a plateau level seperation of .033 in $1/Log(j_{trns})$ will be used here.

The results are shown in Fig. 4.3, where two extrapolated plateaus are

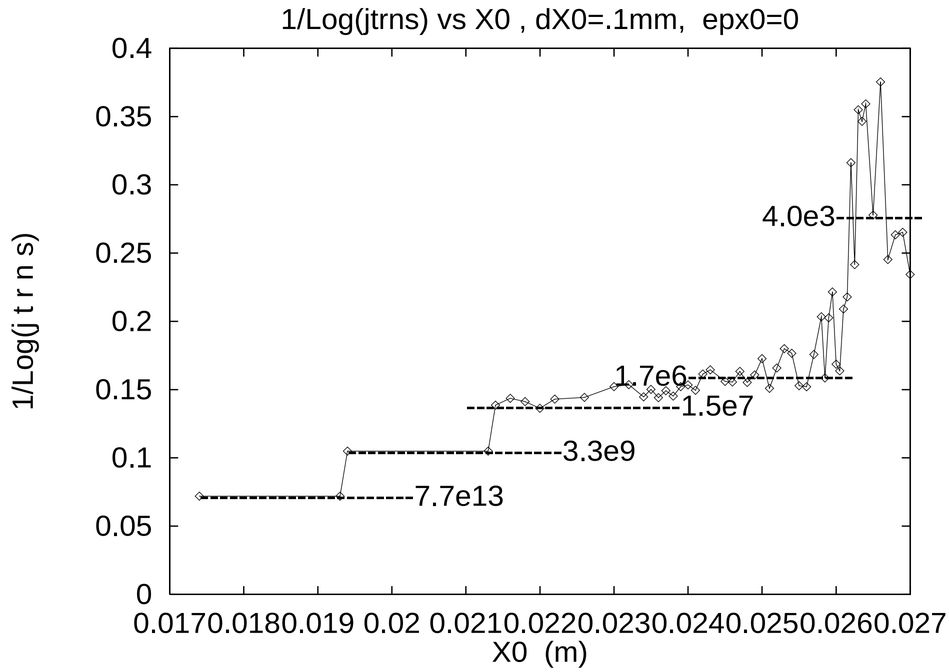

Figure 4.3: $1/Log(j_{trns})$ versus X_0 . $dp/p = 0$, $p_{x0} = 0, p_{y0} = 0, \epsilon_{x0} = 0$ direction, dX_0=.1 mm, $X_0 = (bex0/bey0)^{.5}y_0$, showing extrapolated plateaus. In the figure, 1/ Log(jtrns), X0 , dX0, epx0, epy0 represent $1/Log(j_{trns})$, X_0, dX_0 ,ϵ_{x0}, ϵ_{y0}.

shown with the plateau levels 3.3e9, and 7.7e13 turns . Most of the particles on the 3.3e9 plateau will be assumed to survive 1e9 turns, and the aperture for 1e9 turns is then 21.4mm. 21.4mm is to be compared with the aperture of 24.9 mm found with runs of 1e6 turns. A loss of 3.5 mm or 17%.

Chapter 5

Extrapolation parameters for the plateau model

direction in phase space	ϵ_{y0}=ϵ_{x0}	ϵ_{y0}=0	ϵ_{x0}=0
plateau level, j_{trns} plateau level, $1/Log(j_{trns})$ platea level seperation in $1/Log(j_{trns})$	15e6,1.5e5,2e4 .139, .193, .232 .054, .039	8e6,1.5e6,1e4 .145, .162 .017	1.8e7,1.7e6,4e3 .138,.160,.277 .022
average plateau seperation in $1/Log(j_{trns})$	.033		
plateau width, ΔX_0 (mm) plateau width, $\Delta X_0/X_0$	1.70, 1.83, 2.38 .075,.051,.106	2.0, 1.8 .095,.086	2.2+, 2.0 .092, .083
average plateau width, ΔX_0	2.00 mm		

Table 5.1: Plateau parameters for three directions in phase space. $X_0 = (\beta_{x0}(\epsilon_{x0} + \epsilon_{y0}))^{.5}$.

The extrapolation depends on two parameters, the width of the plateaus and the seperation between consecutive plateau levels as measured in terms of $1/Log(j_{trns})$. The behaviour of these two parameters was studied by doing tracking runs for 3 different cases, corresponding to the three different directions in phase space, ϵ_{y0}=0, ϵ_{x0}=ϵ_{y0} and ϵ_{x0}=0. The two parameters were measured for these 3 cases using runs of about 2e7 turns. The results are summarized in Table 5.1. Altogether, 7 plateaus were found and the two parameters for these 7 plateaus were measured. In Table 5.1 one sees that the plateau width, as measured as ΔX_0, $X_0 = (\beta_{x0}(\epsilon_{x0} + \epsilon_{y0}))^{.5}$, is relatively

constant with an average value of ΔX_0=2.00 mm. The plateau level seperation when measured in $1/Log(j_{trns})$ varies considerably with an average value of .033.

Based on the above results, it is proposed that in extrapolating the survival function, it is assumed that the extrapolated plateau widths are given by ΔX_0=2.00 mm, and the plateau level seperations in $1/Log(j_{trns})$ are .033.

Chapter 6

Conclusions

Tracking studies lead to a model of the survival function, which pictures it as sequence of plateaus. Within the plateaus, the survival time in turns, j_{trns}, oscillates about some constant vaue of j_{trns} which will be called the level of the plateau. Studying the survival function along different directions in phase space, using an older version of the RHIC lattice , one finds that the width of the plateaus, ΔX_0, $X_0 = (\beta_{x0}(\epsilon_{x0} + \epsilon_{y0}))^{.5}$, remains roughly constant at about 2.00 mm. The seperation between the levels of adjacent plateaus has the same order of magnitude when measured in terms of the change in $1/Log(j_{trns})$, and has an average vaue of .033. Using, these results for the width of the plateaus and the seperation between plateau levels, one can extrapolate to estimate the location of the plateaus that correspond to longer survival times than can be found by tracking. For the case treated, it was found that a required survival time of 1e9 turns reduced the aperture by about 15% as compared to the aperture found by tracking using 1e6 turns.

The plateau model also leads to new criteria to be used in tracking studies to find the aperture for particles to survive a given number of turns. In the plateau model, one finds the first plateau whose level is higher than the given number of turns, in order to find the aperture for the given number of turns. This is to be compared with often used method, where one does a search starting at large amplitudes until one finds an amplitude that survives the given number of turns. In the latter method one cannot be sure that a finer search would not find unstable runs at smaller amplitudes or how frequently these unstable runs will occur. In the plateau model, while there may be unstable runs at smaller amplitudes, there is the assumption that they will not occur frequently.

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